2023 Advanced Manufacturing & Industrial AI Qualify Exam

• The examination will be four hours long.

• There will be six questions in all. Students must select 4 out of 6 questions to answer.

• The exam is open book and open notes. The students can bring any relevant written materials.

• No laptop, tablet, and internet access are allowed.

• Calculator is allowed.

Problem 1 (ISyE 510)

A manufacturing cell has four departments, labeled as A, B, C, D respectively. Four products will be produced by this cell. The daily production rate is given by Table 1. These four departments will be located at four locations, labeled as 1, 2, 3, 4. The distances among these locations are given by Table 2.

Table 1. Production sequence and daily rate

Product type	Processing sequence	Daily production
1	C B D A	150
2	D B	200
3	D C A	250
4	D B A C B	300

Table 2. Distance among locations

	1	2	3	4
1	-	80	10	30
2	50	-	20	70
3	90	60	-	40
4	75	60	45	-

Please answer the following questions:

(a) If we put A, B, C, D at locations 1, 2, 3, 4, respectively, what is the overall material handling cost?
(b) What is the material handling cost if we switch the location of department A and D?
(c) If we put department D at location 2, what is the possible minimum material handling cost for the materials flow-in and flow-out of D?
(d) Given the following matrix, use Hungarian method to find four numbers whose summation is minimized while each one uniquely occupy one row and one column of the matrix. **Please note that this matrix is not related with the problem (a)~(c).**

92	80	53	60
63	44	55	54
88	98	28	84
64	10	37	15

Problem 2 (ISyE 512)

A beautiful machine came to a bad end on April 20, 2023, when SpaceX's 40-story tall Starship rocket consumed itself in an orange and white fireball just four minutes after launch and 39 km (24 mi.) above the Gulf of Mexico off the coast of Texas.

5 out of 33 Raptor rocket engines failed during the launch, leading to this catastrophic failure.

Fig. 1 (left-right): 1st Generation Starship, Raptor Rocket Engines, Engine Failures, Explosion

As a quality engineer, you are assigned to look into the 2nd generation starship (total 33 engines).

(1) Suppose the failure of an engine occurs at random, following a Poisson distribution with parameter $\lambda = 0.04$, what is the probability that the 2nd generation Starship will have two or more failed engines in the next launch?

(2) In the production of 2nd generation engines, two inspectors sampled 10 engines. They use the same gauge to measure the diameter of engines three times each. The USL and LSL are given as 5.0 ± 1.0 inch. The averages and ranges of the measurements on each engine are given below (units are in inches). The total observed variance (σ^2_{total}) is 0.56-inch. What's the estimated variability of engines? What's the estimated σ_{gauge}? Can you assess the capability of the measurement system?

(3) Finally, you are asked to establish a control chart for the overall engine production process. This production process continuously manufactures engines. Assuming the inspection unit is one engine, and a common chart for nonconformities is to be used. As a preliminary data, 9 nonconformities were counted when inspecting 300 engines. Can you establish a two-sigma control limits? Can you find out α error for the control chart? Can you find the β error for the chart with two-sigma control limits if the average number of defects is actually one (i.e., c=1)?

Sampled Engine Number	Inspector 1		Inspector 2	
	$\bar{x}$	R	$\bar{x}$	R
1	14.9/3	0.1	14.9/3	0.3
2	15.5/3	0.1	15.3/3	0.0
3	15.3/3	0.3	15.7/3	0.3
4	15.0/3	0.2	14.9/3	0.3
5	14.5/3	0.1	14.5/3	0.1
6	15.2/3	0.2	15.2/3	0.2
7	15.3/3	0.0	15.1/3	0.1
8	15.1/3	0.3	15.1/3	0.5
9	15.1/3	0.1	14.8/3	0.3
10	14.2/3	0.3	14.1/3	0.2
Average	**15.01/3**	**0.17**	**14.96/3**	**0.23**

Problem 3 (ISyE 605)

Part A.

Consider 4 jobs J1, J2, J3, and J4 received at day 0 on the production calendar. Each of these jobs needs two steps to be finished. Two workstations A and B are used to handle these two steps, respectively. The processing time for each of these two steps are listed below:

Jobs	J1	J2	J3	J4
Step 1 processing time (day) at workstation A	4	7	3	15
Step 2 processing time (day) at workstation B	11	7	9	8

Assume the job sequences are the same on A and B in the following problems.

(1) If the processing sequence of these jobs is J1 -> J2 -> J3 -> J4, what is the makespan of these four jobs?
(2) What is the mean flow time of these jobs in (1)?
(3) If we only consider Step 1 and ignore Step 2, then what is the optimal processing sequence to minimize the mean flow time of the jobs passing Step 1?
(4) If we consider both Step 1 and Step2, please find the optimal processing sequence that minimize the makespan of these 4 jobs.

Part B.

If we have 10 control points, labeled as $\mathbf{P}_0$ to $\mathbf{P}_9$, then

(1) What are the non-periodic knot values for the 2nd order B-spline curve?
(2) Find the blending function for control point $\mathbf{P}_5$, i.e., if the curve is expressed as

$$\mathbf{r}(u) = \sum_{i=0}^{9} N_{i,2}(u)\mathbf{P}_i$$, then find $N_{5,2}(u)$.

Problem 4 (ISyE 641)

The table below shows the average number of jobs per week that flow between different work areas within a manufacturing facility. The facility produces a wide variety of parts from sheet metal and plate metal. The flow is from the work area indicated by the row and to the work area indicated by the column.

		A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**
A	Laser 1	--	0	0	0	185	0	15	37	0
B	Laser 2	0	--	0	0	90	55	18	24	3
C	Turret Punch Press	0	0	--	0	15	79	12	36	7
D	Water Jet	0	0	0	--	28	68	7	16	4
E	Fab Cell 1	0	0	0	0	--	0	26	204	45
F	Fab Cell 2	0	0	0	0	0	--	118	145	23
G	Painting	0	0	0	0	0	0	--	0	138
H	Welding	0	0	0	0	0	0	0	--	301
I	Packing & Shipping	0	0	0	0	0	0	0	0	--

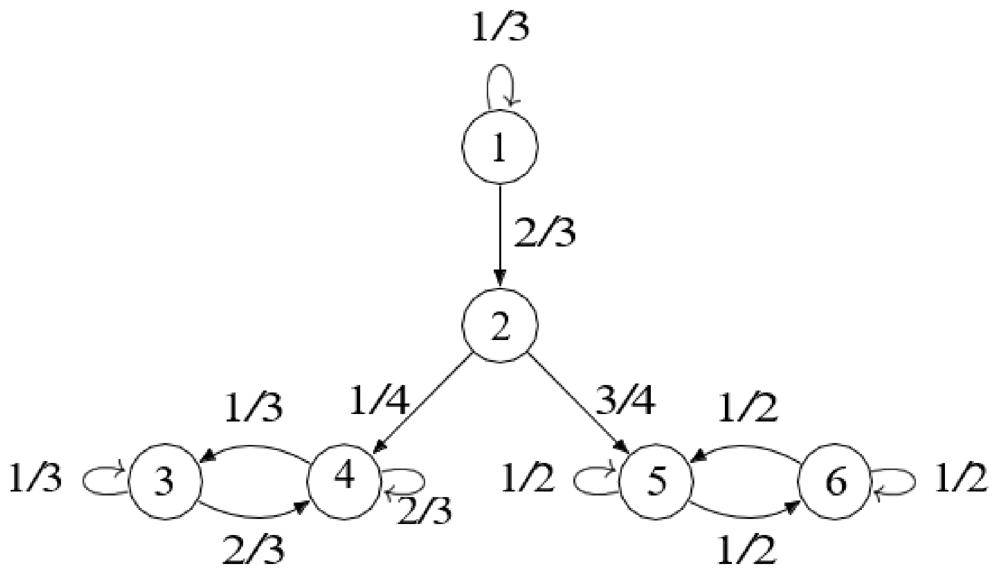
a) Propose a POLCA production control system for this facility. How many loops will there be, and what are the loops?

b) Do you think POLCA seems logical for this scenario? Explain why or why not.

c) What are the next steps that need to be accomplished if the company moves forward with POLCA?

Problem 5 (ISyE624)

A discrete time Markov chain $X_n, n \geq 0$ with the state space $\{1, 2, 3, 4, 5, 6\}$ has the following transition diagram:



Transition probabilities are given in the diagram next to corresponding transition. (E.g. $p(4, 3) = 1/3$, $p(4, 4) = 2/3$.)

(a) Find the recurrent and transient states of the Markov chain. (Make sure to provide justification!)

(b) Show that $\lim_{n\to\infty} P(X_n = 3 \mid X_0 = 4)$ exists and find its value.

(c) Suppose $X_0 = 2$. What is the long-term fraction of time that X_n spends in the set $\{1, 2, 3\}$?

Problem 6 (ISYE 645)

Suppose that in the newsvendor problem, the demand per period, D, has a Poisson distribution with mean λ. Let $F(\cdot)$ and $f(\cdot)$ be the CDF and PMF of demand distribution. Specifically, we have $f(k) = \lambda^k e^{-\lambda}/k!$. Note that h is the unit holding (or overage) cost, p represents the unit stockout (or underage) cost. The total cost is given by

$$g(S) = \mathbb{E}_d[hI(S,d) + pB(S,d)],$$

where $I(S,d) = [S-d]^+$ and $B(S,d) = [d-S]^+$ are the leftover and shortage, respectively. Suppose further that there exists an optimal base-stock level S^* such that $F(S^*) = p/(h+p)$.

Question: Prove that

$$g(S^*) = (h+p)f(S^*)\lambda$$

[A derivative table is attached in case.]

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(f(x) + g(x)) = f'(x) + g'(x)$$

$$\frac{d}{dx}(f(x)\,g(x)) = f'(x)\,g(x) + f(x)\,g'(x)$$

$$\frac{d}{dx}(x^n) = nx^{n-1}, \text{ for real numbers } n$$

$$\frac{d}{dx}(cf(x)) = cf'(x)$$

$$\frac{d}{dx}(f(x) - g(x)) = f'(x) - g'(x)$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$$

Appendix

Appendix VI Factors for Constructing Variables Control Charts

Observations in Sample, n	Chart for Averages: Factors for Control Limits A	A_2	A_3	Chart for Standard Deviations: Factors for Center Line c_4	$1/c_4$	Factors for Control Limits B_3	B_4	B_5	B_6	Chart for Ranges: Factors for Center Line d_2	$1/d_2$	d_3	Factors for Control Limits D_1	D_2	D_3	D_4
2	2.121	1.880	2.659	0.7979	1.2533	0	3.267	0	2.606	1.128	0.8865	0.853	0	3.686	0	3.267
3	1.732	1.023	1.954	0.8862	1.1284	0	2.568	0	2.276	1.693	0.5907	0.888	0	4.358	0	2.575
4	1.500	0.729	1.628	0.9213	1.0854	0	2.266	0	2.088	2.059	0.4857	0.880	0	4.698	0	2.282
5	1.342	0.577	1.427	0.9400	1.0638	0	2.089	0	1.964	2.326	0.4299	0.864	0	4.918	0	2.115
6	1.225	0.483	1.287	0.9515	1.0510	0.030	1.970	0.029	1.874	2.534	0.3946	0.848	0	5.078	0	2.004
7	1.134	0.419	1.182	0.9594	1.0423	0.118	1.882	0.113	1.806	2.704	0.3698	0.833	0.204	5.204	0.076	1.924
8	1.061	0.373	1.099	0.9650	1.0363	0.185	1.815	0.179	1.751	2.847	0.3512	0.820	0.388	5.306	0.136	1.864
9	1.000	0.337	1.032	0.9693	1.0317	0.239	1.761	0.232	1.707	2.970	0.3367	0.808	0.547	5.393	0.184	1.816
10	0.949	0.308	0.975	0.9727	1.0281	0.284	1.716	0.276	1.669	3.078	0.3249	0.797	0.687	5.469	0.223	1.777
11	0.905	0.285	0.927	0.9754	1.0252	0.321	1.679	0.313	1.637	3.173	0.3152	0.787	0.811	5.535	0.256	1.744
12	0.866	0.266	0.886	0.9776	1.0229	0.354	1.646	0.346	1.610	3.258	0.3069	0.778	0.922	5.594	0.283	1.717
13	0.832	0.249	0.850	0.9794	1.0210	0.382	1.618	0.374	1.585	3.336	0.2998	0.770	1.025	5.647	0.307	1.693
14	0.802	0.235	0.817	0.9810	1.0194	0.406	1.594	0.399	1.563	3.407	0.2935	0.763	1.118	5.696	0.328	1.672
15	0.775	0.223	0.789	0.9823	1.0180	0.428	1.572	0.421	1.544	3.472	0.2880	0.756	1.203	5.741	0.347	1.653
16	0.750	0.212	0.763	0.9835	1.0168	0.448	1.552	0.440	1.526	3.532	0.2831	0.750	1.282	5.782	0.363	1.637
17	0.728	0.203	0.739	0.9845	1.0157	0.466	1.534	0.458	1.511	3.588	0.2787	0.744	1.356	5.820	0.378	1.622
18	0.707	0.194	0.718	0.9854	1.0148	0.482	1.518	0.475	1.496	3.640	0.2747	0.739	1.424	5.856	0.391	1.608
19	0.688	0.187	0.698	0.9862	1.0140	0.497	1.503	0.490	1.483	3.689	0.2711	0.734	1.487	5.891	0.403	1.597
20	0.671	0.180	0.680	0.9869	1.0133	0.510	1.490	0.504	1.470	3.735	0.2677	0.729	1.549	5.921	0.415	1.585
21	0.655	0.173	0.663	0.9876	1.0126	0.523	1.477	0.516	1.459	3.778	0.2647	0.724	1.605	5.951	0.425	1.575
22	0.640	0.167	0.647	0.9882	1.0119	0.534	1.466	0.528	1.448	3.819	0.2618	0.720	1.659	5.979	0.434	1.566
23	0.626	0.162	0.633	0.9887	1.0114	0.545	1.455	0.539	1.438	3.858	0.2592	0.716	1.710	6.006	0.443	1.557
24	0.612	0.157	0.619	0.9892	1.0109	0.555	1.445	0.549	1.429	3.895	0.2567	0.712	1.759	6.031	0.451	1.548
25	0.600	0.153	0.606	0.9896	1.0105	0.565	1.435	0.559	1.420	3.931	0.2544	0.708	1.806	6.056	0.459	1.541

For $n > 25$

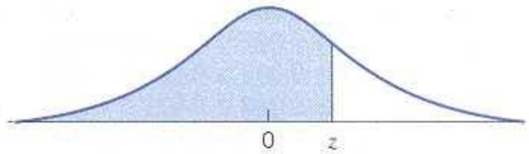
$$A = \frac{3}{\sqrt{n}}, \quad A_3 = \frac{3}{c_4\sqrt{n}}, \quad c_4 \simeq \frac{4(n-1)}{4n-3},$$

$$B_3 = 1 - \frac{3}{c_4\sqrt{2(n-1)}}, \quad B_4 = 1 + \frac{3}{c_4\sqrt{2(n-1)}},$$

$$B_5 = c_4 - \frac{3}{\sqrt{2(n-1)}}, \quad B_6 = c_4 + \frac{3}{\sqrt{2(n-1)}}.$$

Appendix II Cumulative Standard Normal Distribution

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-u^2/2}\, du$$



z	0.00	0.01	0.02	0.03	0.04	z
0.0	0.50000	0.50399	0.50798	0.51197	0.51595	0.0
0.1	0.53983	0.54379	0.54776	0.55172	0.55567	0.1
0.2	0.57926	0.58317	0.58706	0.59095	0.59483	0.2
0.3	0.61791	0.62172	0.62551	0.62930	0.63307	0.3
0.4	0.65542	0.65910	0.62276	0.66640	0.67003	0.4
0.5	0.69146	0.69497	0.69847	0.70194	0.70540	0.5
0.6	0.72575	0.72907	0.73237	0.73565	0.73891	0.6
0.7	0.75803	0.76115	0.76424	0.76730	0.77035	0.7
0.8	0.78814	0.79103	0.79389	0.79673	0.79954	0.8
0.9	0.81594	0.81859	0.82121	0.82381	0.82639	0.9
1.0	0.84134	0.84375	0.84613	0.84849	0.85083	1.0
1.1	0.86433	0.86650	0.86864	0.87076	0.87285	1.1
1.2	0.88493	0.88686	0.88877	0.89065	0.89251	1.2
1.3	0.90320	0.90490	0.90658	0.90824	0.90988	1.3
1.4	0.91924	0.92073	0.92219	0.92364	0.92506	1.4
1.5	0.93319	0.93448	0.93574	0.93699	0.93822	1.5
1.6	0.94520	0.94630	0.94738	0.94845	0.94950	1.6
1.7	0.95543	0.95637	0.95728	0.95818	0.95907	1.7
1.8	0.96407	0.96485	0.96562	0.96637	0.96711	1.8
1.9	0.97128	0.97193	0.97257	0.97320	0.97381	1.9
2.0	0.97725	0.97778	0.97831	0.97882	0.97932	2.0
2.1	0.98214	0.98257	0.98300	0.98341	0.98382	2.1
2.2	0.98610	0.98645	0.98679	0.98713	0.98745	2.2
2.3	0.98928	0.98956	0.98983	0.99010	0.99036	2.3
2.4	0.99180	0.99202	0.99224	0.99245	0.99266	2.4
2.5	0.99379	0.99396	0.99413	0.99430	0.99446	2.5
2.6	0.99534	0.99547	0.99560	0.99573	0.99585	2.6
2.7	0.99653	0.99664	0.99674	0.99683	0.99693	2.7
2.8	0.99744	0.99752	0.99760	0.99767	0.99774	2.8
2.9	0.99813	0.99819	0.99825	0.99831	0.99836	2.9
3.0	0.99865	0.99869	0.99874	0.99878	0.99882	3.0
3.1	0.99903	0.99906	0.99910	0.99913	0.99916	3.1
3.2	0.99931	0.99934	0.99936	0.99938	0.99940	3.2
3.3	0.99952	0.99953	0.99955	0.99957	0.99958	3.3
3.4	0.99966	0.99968	0.99969	0.99970	0.99971	3.4
3.5	0.99977	0.99978	0.99978	0.99979	0.99980	3.5
3.6	0.99984	0.99985	0.99985	0.99986	0.99986	3.6
3.7	0.99989	0.99990	0.99990	0.99990	0.99991	3.7
3.8	0.99993	0.99993	0.99993	0.99994	0.99994	3.8
3.9	0.99995	0.99995	0.99996	0.99996	0.99996	3.9

$$\Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-u^2/2}\, du$$

z	0.05	0.06	0.07	0.08	0.09	z
0.0	0.51994	0.52392	0.52790	0.53188	0.53586	0.0
0.1	0.55962	0.56356	0.56749	0.57142	0.57534	0.1
0.2	0.59871	0.60257	0.60642	0.61026	0.61409	0.2
0.3	0.63683	0.64058	0.64431	0.64803	0.65173	0.3
0.4	0.67364	0.67724	0.68082	0.68438	0.68793	0.4
0.5	0.70884	0.71226	0.71566	0.71904	0.72240	0.5
0.6	0.74215	0.74537	0.74857	0.75175	0.75490	0.6
0.7	0.77337	0.77637	0.77935	0.78230	0.78523	0.7
0.8	0.80234	0.80510	0.80785	0.81057	0.81327	0.8
0.9	0.82894	0.83147	0.83397	0.83646	0.83891	0.9
1.0	0.85314	0.85543	0.85769	0.85993	0.86214	1.0
1.1	0.87493	0.87697	0.87900	0.88100	0.88297	1.1
1.2	0.89435	0.89616	0.89796	0.89973	0.90147	1.2
1.3	0.91149	0.91308	0.91465	0.91621	0.91773	1.3
1.4	0.92647	0.92785	0.92922	0.93056	0.93189	1.4
1.5	0.93943	0.94062	0.94179	0.94295	0.94408	1.5
1.6	0.95053	0.95154	0.95254	0.95352	0.95448	1.6
1.7	0.95994	0.96080	0.96164	0.96246	0.96327	1.7
1.8	0.96784	0.96856	0.96926	0.96995	0.97062	1.8
1.9	0.97441	0.97500	0.97558	0.97615	0.97670	1.9
2.0	0.97982	0.98030	0.98077	0.98124	0.98169	2.0
2.1	0.98422	0.98461	0.98500	0.98537	0.98574	2.1
2.2	0.98778	0.98809	0.98840	0.98870	0.98899	2.2
2.3	0.99061	0.99086	0.99111	0.99134	0.99158	2.3
2.4	0.99286	0.99305	0.99324	0.99343	0.99361	2.4
2.5	0.99461	0.99477	0.99492	0.99506	0.99520	2.5
2.6	0.99598	0.99609	0.99621	0.99632	0.99643	2.6
2.7	0.99702	0.99711	0.99720	0.99728	0.99736	2.7
2.8	0.99781	0.99788	0.99795	0.99801	0.99807	2.8
2.9	0.99841	0.99846	0.99851	0.99856	0.99861	2.9
3.0	0.99886	0.99889	0.99893	0.99897	0.99900	3.0
3.1	0.99918	0.99921	0.99924	0.99926	0.99929	3.1
3.2	0.99942	0.99944	0.99946	0.99948	0.99950	3.2
3.3	0.99960	0.99961	0.99962	0.99964	0.99965	3.3
3.4	0.99972	0.99973	0.99974	0.99975	0.99976	3.4
3.5	0.99981	0.99981	0.99982	0.99983	0.99983	3.5
3.6	0.99987	0.99987	0.99988	0.99988	0.99989	3.6
3.7	0.99991	0.99992	0.99992	0.99992	0.99992	3.7
3.8	0.99994	0.99994	0.99995	0.99995	0.99995	3.8
3.9	0.99996	0.99996	0.99996	0.99997	0.99997	3.9